Spins-Orbit Interactions In Concentric Double Quantum Rings

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ABSTRACT

We investigate the electronic and spin properties of a two-dimensional concentric double quantum rings in the presence of both the Rashba and Dresselhaus spin-orbit interactions and a magnetic field perpendicular to the ring plane. It is shown that the Zeeman and spin-orbit interactions decrease the energy of levels and also lift the spin degeneracy of states. The interplay between the Rashba and Dresselhaus spin-orbit interactions are investigated. In the presence of the spin-orbit interactions, the effects of inter-ring coupling are explored by changing the confinement strength of the confinement potential. The ground state charge and spin densities are calculated in the presence of spin-orbit interactions and in the absence of magnetic field. It is found that the spin-orbit interactions distribute the spin-up and down densities asymmetrically. Then, we have a net nonzero value of the spin which indicates the system is spin-polarized.

Keywords- Spin-orbit interaction, Quantum rings, Quantum confinement, Rashba effect, Dresselhaus effect, semiconductor nanostructures

INTRODUCTION

In recent years, the spin-orbit interaction (SOI) in semiconductor nanostructures has generated great interest from both academic and practical perspectives [1]. The SOI allows to manipulate the spin of electron and could be of use in spintronic and quantum computing. There are two types of SOI in conventional semiconductors. One is the Dresselhaus spin-orbit interaction (DSOI) induced by bulk inversion asymmetry [2] and the other is the Rashba spin-orbit interaction (RSOI) due to structure inversion asymmetry [3]. The RSOI is electrically tunable with a gate electrode or asymmetric doping.

During the last ten years, with the great progress in experimental methods, it is possible to fabricate high quality semiconductor quantum rings with a few electrons [4]. Quantum rings (QRs), due to their unique topologies, display various pure quantum effects such as the Aharonov-Bohm oscillations [5] and persistent current [6]. QRs are considered “artificial atoms” because they have discrete energy levels and shell
structure [7]. The coupled geometry of QRs are called artificial molecule because the inter-ring coupling causes the localized states in the individual rings hybridize forming molecular-like states. Recently, the concentric double quantum rings (CDQRs) have been fabricated [8]. Although the studies involving CDQRs have already generated a wealth of literature (see [9-13] and references therein), a few works have been done on the effects of the spin-orbit interactions in CDQRs [15-17]. The magneto-optical transitions in a semiconductor double ring in the presence of RSOI and magnetic flux have been studied numerically by Kuan et. al. [14]. They have found that the SOI has important influence on the occurrence of level crossings, showing the evidence both for the periodic orbital motion and for spin flips. The spin-dependent electron transport properties have been studied in an one-dimensional double quantum rings in the presence of RSOI using quantum waveguide theory [15-16]. The transmission coefficient is calculated and the effects of RSOI, ring size and magnetic flux on the transmission coefficient have been investigated. In Ref. [15], it has been found that the number of the transmission coefficient peaks is related to the length ratio between the upper arm and the half circumference of the ring. The other study has shown that the system acts as a spin polarizer and spin-inverter (or NOT spin gate) in the particular region of the RSOI [16]. However, as far as we know, only one study has explored the spin-orbit interaction in an isolated two-dimensional CDQRs. In addition, in all studies on the double quantum rings, the RSOI was only considered and the effect of the DSOI is ignored. The aim of this paper is to study the electronic and spin properties of a CDQRs in the presence of the RSOI, DSOI and a magnetic field perpendicular to the CDQRs plane by using two-dimensional finite difference method. The interplay of RSOI and DSOI in a CDQRs are also investigated.

**THEORETICAL MODEL**

Consider a two-dimensional CDQRs with a confinement potential as shown in Fig. 1. In the presence of a magnetic field applied perpendicularly to the CDQRs plane (i.e., $B = Be_z$), the Hamiltonian of a single electron can be written as

$$H = \left(\frac{\mathbf{p}^2}{2m^*} + V(r)\right)\mathbf{1}_1 + \frac{1}{2}g\mu_B\sigma_z + H_R + H_D \mathbf{1}_1$$

(1)

where $m^*$ is the effective electron mass, $g$ stands for the Landé factor, $\mu_B$ is the Bohr magneton and $H_R$ and $H_D$ are the contribution of the linear Rashba and Dresselhaus spin-orbit interactions, respectively. In addition,

$$V(r) = \frac{1}{2}m^*\omega^2 \min((r-r_1)^2,(r-r_2)^2)$$

(2)

is the lateral confinement potential of the CDQRs. Here, $\omega$ is the confinement potential strength, $r_1$ and $r_2$ are the radius of the internal and external rings, respectively.

For symmetric gauge $A = \frac{1}{2}B r e_\phi$. 
\[ P = -i \hbar \nabla + eA = -i \frac{\hbar}{\hbar} \varphi + e Br + \frac{\hbar}{2} \varphi + \left( \frac{\hbar}{r} \varphi + \frac{B \pi r^2}{\hbar} \varphi \right) \]  

(3)

and thus

\[ H_R = \frac{\hbar}{\hbar} (p_x \sigma_x - p_y \sigma_y) = i \alpha (-\sin \varphi \sigma_x + \cos \varphi \sigma_y) \frac{\varphi}{\hbar} - \frac{1}{\hbar} (\cos \varphi \sigma_x + \sin \varphi \sigma_y) \left( \frac{\varphi}{\hbar} - \frac{B \pi r^2}{\hbar} \right), \]

(4)

\[ H_D = \frac{\hbar}{\hbar} (p_x \sigma_x - p_y \sigma_y) = i \beta (-\cos \varphi \sigma_x + \sin \varphi \sigma_y) \frac{\varphi}{\hbar} + \frac{1}{\hbar} (\sin \varphi \sigma_x + \cos \varphi \sigma_y) \left( \frac{\varphi}{\hbar} + \frac{B \pi r^2}{\hbar} \right), \]

(5)

where, \( \hbar \) is the reduced Planck constant, \( \hbar \) is the flux quantum, \( \sigma_x, \sigma_y \) and \( \sigma_z \) are the Pauli matrices, \( r \) and \( \varphi \) are the radial and angular components in polar coordinates, \( \alpha \) and \( \beta \) are the Rashba and Dresselhaus coupling constants, respectively.

\[ \text{Fig. 1 Schematic view of a parabolic confinement potential for a coupled concentric double quantum rings} \]

In the basis of \( \sigma_z \) eigenfunctions, the Hamiltonian operator (i.e., \( H \)) is a \( 2 \times 2 \) matrix which its elements in the operator form, are as follows [17]:

\[ H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}, \]

(6)

where,

\[ H_{11} = -\frac{\hbar^2}{2m^*} \left( \frac{\varphi}{\varphi^2} + \frac{1}{4r^2} - \frac{1}{r^2} \left( i \frac{\varphi}{\varphi^2} - \frac{B \pi r^2}{\varphi^2} \right)^2 \right) + V(r) + \frac{1}{2} gB \mu_B, \]

(7)

\[ H_{22} = H_{11} - gB \mu_B, \]

(8)

\[ H_{12} = -\frac{1}{r} (\alpha e^{-i \varphi} + i \beta e^{i \varphi})(i \frac{\varphi}{\varphi^2} - \frac{B \pi r^2}{\varphi^2}) + (\alpha e^{-i \varphi} - i \beta e^{i \varphi})(-\frac{1}{2r} + \frac{\varphi}{\varphi^2} \right), \]

(9)
\begin{equation}
H_{21} = \frac{1}{r}(-\alpha e^{i\varphi} + i \beta e^{-i\varphi})(i \frac{\partial}{\partial \varphi} - B \frac{\pi r^2}{\phi_0} - (\alpha e^{i\varphi} + i \beta e^{-i\varphi})(\frac{1}{2r} + \frac{\partial}{\partial r}),
\end{equation}

where \( H_{12} = H_{21} \). The eigenfunctions of \( H \) can be split as the sum of two spin-polarized spatial wave functions,

\begin{equation}
\psi = \psi_\uparrow \chi_+ + \psi_\downarrow \chi_-,
\end{equation}

where \( \chi_+ \) and \( \chi_- \) are the \( \sigma_z \) eigenfunctions. The Schrödinger equation derived from the Hamiltonian [Eq. (6)] is discretized by using the finite difference method in polar coordinates \( r \) and \( \varphi \). This yields an eigenvalue problem with a large asymmetric sparse matrix which is solved by employing the iterative Arnoldi factorization [18].

**RESULTS AND DISCUSSION**

For our calculations, we consider the parameters corresponding to etched InGaAs/GaAs materials [19]:

\( m^* = 0.063m_0, \beta = 10.8\text{meV nm} \). This value is obtained from the bulk Dresselhaus constant (\( \beta_b \)) as \( \beta = (\pi^2 / d) \beta_b \), where \( d = 5\text{nm} \) is the height of the structure in the growth direction and \( \beta_b = 27.5\text{eV \AA}^3 \) for GaAs [20]. We set \( g = -2.15 \) when the Zeeman effect is present. The confinement strength of the CDQRs is chosen \( \hbar \omega = 3\text{meV} \) and the radius of internal and external rings are considered \( r_i = 50\text{nm} \) and \( r_e = 100\text{nm} \), respectively. In Fig. 2, the single-electron energy spectrum is plotted as a function of magnetic field in the presence (the red curves) and the absence (the blue curves) of the spin-orbit and Zeeman interactions. It is shown that the spin-orbit and Zeeman interactions decreases the energy of levels and also lift the spin degeneracy of states.

![Fig. 2](image-url)

**Fig. 2** (Color online) Single-electron energy spectrum in the presence of the Rashba and Dresselhaus spin-orbit interactions with \( \alpha = \beta = 10.8\text{meV nm} \) and the Zeeman effect with \( g = -2.15 \) (red curves) and in the absence of them for \( \alpha = \beta = g = 0 \) (blue curves)
Increasing of the RSOI strength to the twice value of the DSOI strength causes the energy of levels more decreases and the anticrossing and spin splitting of levels increases significantly. This situation is shown in Fig. 3. In this figure, the energy levels of the equal strengths (i.e., $\alpha = \beta = 10.8 \text{meV nm}$) of RSOI and DSOI (the red curves) are also shown for comparison.

**Fig. 3** (Color online) Single-electron energy spectrum in the presence of the spin-orbit and Zeeman interactions with $\alpha = \beta = 10.8 \text{meV nm}$, $g = -2.15$ (red curves) and $\beta = 10.8 \text{meV nm}$, $\alpha = 2\beta$, $g = -2.15$ (blue curves)

Figures 4 and 5 show that if only the RSOI or DSOI is considered the energy spectrum is determined by the Zeeman effect. In other word, in the absence of Zeeman effect, as shown in Fig. 4, the energy spectrum for a constant value of the Rashba and Dresselhaus strengths is equal but in the presence of Zeeman effect the energy spectrum (see, Fig. 5) is different. In Fig. 5, the red (blue) curves indicate the energy levels when only the RSOI with $\alpha = 10.8 \text{meV nm}$ is considered while the blue curves correspond to the presence of only the DSOI with $\beta = 10.8 \text{meV nm}$.

**Fig. 4** (Color online) Single-electron energy spectrum in the absence of Zeeman effect ($g = 0$) and in the presence of only Rashba ($\alpha = 10.8 \text{meV nm}$, $\beta = 0$) or Dresselhaus ($\alpha = 0$, $\beta = 10.8 \text{meV nm}$) effect
It is observed that, in the presence of the Zeeman effect, the RSOI more decreases the energy of lowest levels especially for the larger magnetic fields. Since the RSOI is electrically tunable, the effect of variation of the RSOI strength on the ground state energy is shown in Fig. 6 for the three CDQRs with different confinement strengths. Indeed, the increment of confinement strength enhances the barrier potential between the internal and external rings that causes the electron tunnelling is decreased. As shown in Fig. 6, it is seen that the ground state electron energy decreases by increasing of the RSOI strength. The same behaviour can also be seen for the CDQRs with the higher confinement strengths. It can be deduced that the spin-orbit interaction is dominant potential in comparison with the confining potential. Figure 7 is same as Fig. 6 except the confinement strength is fixed as $h\omega=3meV$ and the applied magnetic field is changed. The ground state energy is reduced by increasing of the RSOI strength, however, the decrement is considerable for higher magnetic field.

**Fig. 6** (Color online) The ground state energy as a function of the Rashba spin-orbit strength for $\beta=10.8meV nm, B=0$ and different confinement strengths
Fig. 7 (Color online) The ground state energy as a function of the Rashba spin-orbit strength for $\beta=10.8\text{meV nm, } h\omega=3\text{meV}$ and different magnetic fields.

The ground-state charge and spin densities are calculated for $B=0$, $\alpha=\beta=10.8\text{meV nm}$. Figures 8(a) and (b) show the probability densities of the ground state electron with spin up and down, respectively. It is seen that the spin-orbit interactions distribute the spin up and down densities asymmetrically. Then, we have a net nonzero value of the spin which indicates the system is spin-polarized. However, the charge density (as shown in Fig. 8(c)) is distributed symmetrically and it is observed that the probability of finding the ground-state electron in the internal ring is higher.

SUMMARY AND CONCLUSION

We have considered an electron in a CDQRs in the presence of the RSOI and DSOI. We assumed a parabolic profile for the confinement potential of CDQRs with two minima in the internal and external rings. The effects of the RSOI and DSOI on the energy spectra and electron charge and spin densities are investigated. The interplay between the RSOI and DSOI are explored. In the presence of the spin-orbit interactions, the effects of inter-ring coupling are studied. It is found that the spin-orbit interactions have important effects on the energy spectra and spin densities. Then, the investigation of spin-orbit interactions are necessary to gain deeper insight into the electronic and spin properties of these systems.

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Fig. 8 The ground state electron spin and charge densities for $B = 0$, $\alpha = \beta = 10.8 \text{meV nm}$, (a) for spin-up density, (b) for spin-down density, and (c) for charge density.

REFERENCES


