Reliability Analysis of Phasor Measurement Unit Using Hidden MARKOV Model

Author
Amaresh Choudhury
Email-amaresh.mitu@gmail.com

ABSTRACT
As modern electric power systems are transforming into smart grids, real time wide area monitoring system (WAMS) has become an essential tool for operation and control. With the increasing applications of WAMS for on-line stability analysis and control in smart grids, phasor measurement unit (PMU) is becoming a key element in wide area measurement system and the consequence of the failure of PMU is very severe and may cause a black out. Therefore reliable operation of PMU is very much essential for smooth functioning of the power system. This thesis is focused mainly on evaluating the reliability of PMU using hidden Markov model. Firstly, the probability of given observation sequence is obtained for the individual modules and PMU as a whole using forward and backward algorithm. Secondly, the optimal state sequence each module passes through is found. Thirdly, the parameters of the hidden Markov model are re-estimated using Baum-Welch algorithm.

Index Terms—Hidden Markov Model (HMM), Phasor Measurement Unit, Reliability, wide area monitoring system (WAMS), Viterbi Algorithm, Baum-Welch Algorithm.

INTRODUCTION
The interest in phasor measurement technology has reached a peak in recent years, as the need for the best estimate of the power system's state is recognized to be a crucial element in improving its performance and its resilience in the face of catastrophic failures. All installations are reaching for a hierarchical Wide-area measurement system (WAMS) so that the measurements obtained from various substations on the system can be collected at central locations from which various monitoring, protection, and control applications can be developed. Wide Area Measurement System(WAMS) [1-2] is the advanced technology used to avoid major regional blackouts as those occurred in North America and Canada in 2003. WAMS facilitates the continuous and synchronous monitoring of power system. Phasor Measurement Unit (PMU) is the key component in WAMS, which provides GPS synchronization, synchronized phasor voltages, currents, frequency and rate of
change of frequency. Advantages of GPS system is that it provides accuracy of 1µs i.e. 0.021° for 50 Hz system and 0.018° for 60 Hz system.

**OBJECTIVE OF THE PAPER**

Reliability Analysis of PMU using Hidden Markov Model:
The probability of given observation sequence is obtained for the individual modules and PMU as a whole using forward algorithm.
The optimal state sequence of each module passes through is found.
The parameters of the hidden Markov model are re-estimated using Baum-Welch algorithm.

**HISTORICAL OVERVIEW**

Phase angles of voltage phasors of power network buses have always been of special interest to power system engineers. It is well known that active (real) power flow in a power line is very nearly proportional to sine of the angle difference between voltages at the two terminals of the line. As many of the planning and operational considerations in a power network are directly concerned with the flow of real power, measuring angle differences across transmission has been of concern for many years. The earliest modern application involving direct measurement of phase angle difference, these systems used LORAN-C, GOES satellite transmissions, and the HBG radio transmissions(in Europe) in order to obtain synchronization of reference time at different locations in a power system. The next level available positive going zero crossing of a phase voltage was used to estimate the local phase angle with respect to the time reference. Using the difference of the measured angles on a common reference at two locations, the phase angle difference between voltages at two buses was established. Measurement accuracies achieved in these systems were of the order of 40 µs. Single phase voltage angles were measured and of course no attempt was made to measure the prevailing voltage phasor magnitude. Neither was any account taken of the harmonics contained in the voltage waveform. These methods of measuring phase angle differences are not suitable for generalization for wide area phasor measurement systems and remain one of a kind system which are no longer in use.
The modern area of phasor measurement technology has its genesis in research conducted on computer relaying of transmission lines. Early work on transmission line relaying with microprocessor based relays showed that the available computer power in those days was barely sufficient to manage the calculations needed to perform all the transmission line relaying functions.
A significant portion of the computations was dedicated to solving six fault loop equations at each sample time in order to determine if any one of the ten types of faults possible on a three phase transmission lines are present. The search for methods which would eliminate the need to solve the six equations finally yielded a new relaying technique which was based on symmetrical component analysis of line voltages and currents. Using symmetrical components and certain quantities derived from them, it was possible to perform all fault calculations with single equation. Efficient algorithms for computing symmetrical components of three phase voltages and currents were described and calculation of positive sequence voltages and currents using the algorithms gave an impetus for the development of modern phasor measurement systems. Positive sequence voltages of a network constitute the state vector of a power system, and it is of fundamental importance in all of power system analysis. The Global Positioning System (GPS) was beginning to be fully deployed around that time. It became clear that this system offered the most effective way of synchronizing power system measurements over great distances. The first prototypes of the modern “phasor measurement units” (PMUs) using GPS were built at Virginia Tech in early 1980s, and two of these prototypes are shown in fig 1. The Prototype PMU units built at Virginia Tech were deployed at a few substations of the Bonneville Power Administration, The American Electric Power Service Corporation, and the New York Power Authority. The first commercial manufacture of PMUs with Virginia Tech collaboration was started by Macrodyne in 1991. At present, a number of manufacturers offer PMUs as a commercial product, and deployment of PMUs on power system is being carried out in earnest in many countries around the world. IEEE published a standard in 1991 governing the format of data files created and transmitted by the PMU. A revised version of the standard was issued in 2005.

Concurrently with the development of PMUs as measurement tools, research was on going on applications of the measurements provided by the PMUs. It can be said now that finally the technology of synchronized phasor measurements has come of age and most modern power systems around the world are in the process of installing wide area measurement systems consisting of the phasor measurement units [2].
BASIC DESCRIPTION OF PMU

One of the most important features of the PMU technology is that the measurements are time stamped with high precision at the source, so that the data transmission speed is no longer a critical parameter in making use of this data. All PMU measurements with the same time stamp are used to infer the state of the power system at the instant defined by the time stamp. The Global positioning system (GPS) has become the method of choice for providing the time tags to the PMU measurements. Remember that PMUs evolved out of the development of the “symmetrical component distance relay”. As shown in fig 2 Analog voltage and current signal obtained from the secondary windings of the current and voltage transformers. All three phase current and voltages are used so that positive sequence measurement can be carried out.

The current and voltage signals are converted to voltages with appropriate shunts or instrument transformers (typically within the range of ± 10 volts)

So that they matched with the requirements of the analog –to- digital converters. The sampling rate chosen for the sampling process dictates the frequency response of the anti-aliasing filters. In most cases these are analog-
type filters with cutoff frequency less than half the sampling frequency in order to satisfy Nyquist criterion. The sampling clock is phase-locked with the GPS clock pulse. Sampling rates have been going up steadily over the years – starting with the rate of 12 samples per cycle of the nominal power frequency in the first PMUs to as high as 96 or 128 samples per cycle. The microprocessor calculates positive sequence estimates of all the current signal, voltage signal, frequency and rate of change of frequency. The time stamp is created from two of the signals derived from the GPS receiver [2]. PMUs evolved out of the development of the “symmetrical component distance relay” [4], [6]. The analog inputs are currents and voltages obtained from the secondary windings of the current and voltage transformers. All three phase currents and voltages are used so that positive-sequence measurement can be carried out. In contrast to a relay, a PMU may have currents in several feeders originating in the substation and voltages belonging to various buses in the substation. The current and voltage signals are converted to voltages with appropriate shunts or instrument transformers \((M_1)\) (typically within the range of ±10 volts) so that they are matched with the requirements of the analog-to-digital converters \((M_3)\). The anti-aliasing filter is present to filter unnecessary disturbance and noise \((M_2)\). These three modules constitutes the Data Collection Module \((M_{123}=M_D)\).

In GPS Module \((M_4)\), a crystal oscillator is used to supply the sampling clock pulses for the Analog /Digital (A/D) converting module and track the Pulse Per Second (PPS) supplied by the GPS receiver to correct the error between PPS and crystal oscillator frequency.

The CPU Module \((M_5)\) calculates positive-sequence estimates of all the current and voltage signals and stamps it with coordinated Universal Time (UTC) supplied by the GPS Module \((M_4)\). From the CPU Module \((M_5)\) data are sent data concentrator (PDC) to Super Data Concentrator (SDC) to control centre through Communication Module \((M_6)\). Power Supply Module \((M_7)\) supplies power to the PMU. This is illustrated in Fig. 3.

![Fig. 3 Modules of Phasor Measurement Unit](image-url)
MARKOV MODEL
A Markov chain is a mathematical model for stochastic systems whose states, discrete or continuous, are governed by a transitional probability. In order for the basic Markov approach to be applicable, the behavior of the system must be characterized by a lack of memory, that is, the future states of a system are independent of all past states except the immediately the preceding one. Therefore the future random behavior of a system only depends on where it is at present, not on where it has been in the past or how it arrived at its present position. In addition, the process must be stationary, sometimes called homogeneous, for the approach to be applicable. This means the behavior of the system must be the same at all points of time irrespective of the point of time being considered, i.e., the probability of making a transition from one given state to another is the same (stationary) at all times in the past and future. It is evident from these two aspects, lack of memory and being stationary, that the Markov approach is applicable to those systems whose behavior can be described by a probability distribution that is characterized by a constant hazard rate, i.e., poisson and exponential distribution, since only if the hazard rate is constant does the probability of making a transition between two states remain constant at all points of time

Markov approach can be used for a wide range of reliability problems including systems that are either non-repairable or repairable and are either series-connected, parallel redundant or standby redundant [7].

Markov Property: The state of the system at time t+1 depends only on the state of the system at time t

HIDDEN MARKOV MODEL
A hidden Markov model (HMM) is a Markov model in which the system being modeled is assumed to be a Markov process with unobserved (hidden) states.

Fig. 4: Hidden Markov Model
In this, reliability analysis of PMU is done by Hidden Markov Model (HMM).

Hidden Markov model (HMM) is a statistical model in which the system being modeled is assumed to be a Markov process with unknown parameters, and the challenge is to determine the hidden parameters from the observable parameters. The extracted model parameters can then be used to perform further analysis, for example for pattern/speech recognition applications. In a hidden Markov model, the states are not directly visible, but variables influenced by the state are visible. Each state has a probability distribution over the
possible output tokens. Also, the state transitions are probabilistic in nature. Therefore, the sequence of tokens generated by an HMM gives some information about the sequence of states. The complete HMM model is denoted as $\lambda = (A,B,\pi)$. Simple Markov model is the deterministic model where as HMM is the probabilistic function of the markov chains. This model has been previously used for many applications such as speech recognition [8,9] bank note recognition etc. and has essentially given fruitful results. Using this method complexity of calculation is reduced. Using earlier methods we use to calculate only steady state probabilities but using this method we can calculate the probabilities at every instance .This method has rich mathematical structure and give more accurate and optimum results than previous methods.

HMM Notation
T=the length of the observation sequence
N=the number of states in the model
M=the number of observation symbols
Q=the states of the Markov process
A=the state transition probability matrix
B=Observation probability Matrix
$\pi$=initial state probability Matrix
O=Observation sequence

A. Characterization of an HMM
$N$, the number of states in the model .These states are hidden ,we denote the individual states as $HS=\{HS_1,\ldots,HS_N\}$

The state transition probability matrix $A=\{a_{ij}\}$
where, $a_{ij}=P[S_{t+1}=HS_j | S_t=HS_i]$
i.e. It represents probability of reaching state $HS_j$ at time $t+1$ being in state $HS_i$ at time $t$. State transition depends only on origin and destination(Markov Property)

The observation probability matrix in state $
B = \{b_j(k)\}$ is N x M,

Where $b_j(k)=P\{observation\ k\ at\ t | state\ HS_j\ at\ St\} 1 \leq j \leq M$

The initial state probability $\pi =\{\pi_i\}$,

Where $\pi_i = P[S_1=HS_i]$,

it represents the probability of being in state $HS_i$ at time t=1

HMM has three parameters $A,B$ and $\pi$ .i.e . it is mathematically represented as $\lambda = (A,B,\pi)$
**Hidden Markov Model Problems**

HMM is used to solve three problems:

Problem 1: Given a model \( \lambda = (A,B,\pi) \) and observation sequence \( O \), find \( P(O|\lambda) \). Here we want to determine the likelihood of the observation sequence \( O \).

Problem 2: Given \( \lambda = (A,B,\pi) \) and \( O \), find an optimal state sequence for the underlying Markov process.

Problem 3: Given an observation sequence \( O \) and \( N \) and \( M \), finds the re-estimated model \( \lambda = (A,B,\pi) \).

**Calculation of Probability of an Observation Sequence for a Given Model:** \( P(O|\lambda) \)

**A) Forward Algorithm**

Given model \( \lambda = (A,B,\pi) \) and observation sequence \( O=(O_1,O_2,...,O_N) \), find \( P(O|\lambda) \).

Forward variable at an instant \( t \) for a state \( i \) is given by

\[
\alpha_t(i) = P(O_1,O_2,...,O_t,S_t=H_S_i|\lambda)
\]

Where
- \( \alpha \) = forward variable
- \( \pi \) = starting probability matrix
- \( b_i(O_t) \) = observation probability matrix
- \( A = a_{ij} \) = state transition probability matrix

**STEP 1**

\[
\alpha_1(i) = \pi_i b_i(O_1)
\]

for \( i = 1,...,N \) and \( t = 1,2,...,T \)

**STEP 2**

\[
\alpha_t(i) = \sum_{j=1}^{m} \alpha_{t-1}(j) a_{ij} b_i(O_t)
\]

for \( i = 1,...,N \)

**STEP 3**

\[
P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i)
\]

**B) Backward Algorithm:**

Backward variable at an instant \( t \) for a given state \( i \) is given by

\[
\beta_t(i) = P(O_{t+1},O_{t+2},...,O_T|S_t=H_S_i,\lambda)
\]

Above Equation represents probability of being in state \( H_S_i \) at an instant \( t \) and observing all the observations from the next instant to the end.
\( \beta = \) Calculation of backward variable  
\( \pi = \) starting probability matrix  
\( b_i(O_t) = \) observation probability matrix  
\( A = a_{ij} = \) state transition probability matrix  
For \( t = 1, \ldots, T \) and \( i = 1, \ldots, N \)  
\( N = 4, M = 4 \)  
STEP 1 Let \( \beta_T(i) = 1 \) for \( i = 1, \ldots, N \)  
for \( t = T-1, T-2, \ldots, 1 \) and \( i = 1, \ldots, N \),  
STEP 2  
\[
\beta_t(i) = \sum_{j=1}^{M} a_{ij} b_j(O_{t+1}) \beta_{t+1}(j)
\]

C) Determination of Gama  
\( \alpha = \) forward variable  
\( \beta = \) backward variable  
P(O|\lambda) = probability of observation sequence  
For \( t = 1, \ldots, T \) and \( i = 1, \ldots, N \)  
\( N = 4, T = 2 \)  
\[
\gamma_{t}(i) = \frac{\alpha_t(i) \cdot \beta_t(i)}{P(O|\lambda)}
\]

FOUR STATE HIDDEN MARKOV MODEL: AN ILLUSTRATION  
B. The probability for a given observation sequence and the possible state sequence satisfying the observation sequence are given as follows  
Given observation sequence (ON OFF)  
The possible state sequences are:  
a) HS1-HS4  
b) HS2-HS4  
c) HS3-HS4  
Probability of a given observation sequence ON OFF is \( P(\text{ON,OFF}) \) and is calculated as follows:  
Probability of the state sequence HS1 – HS4 is
\( P(\text{HS} 1 \text{ HS4}) = \pi_1 b_1(o_1) a_{14} b_4(o_2) \)

Probability of the state sequence HS2 – HS4 is

\( P(\text{HS2 HS4}) = \pi_2 b_2(o_1) a_{24} b_4(o_2) \)

Probability of the state sequence HS3 – HS4 is

\( P(\text{HS3 HS4}) = \pi_3 b_3(o_1) a_{34} b_4(o_2) \)

Given Observation sequence is OFF ON

Probability of given observation sequence (OFF ON) is \( P(\text{OFF,ON}) \)

The possible state sequences are

a) HS4-HS2
b) HS4-HS1
c) HS4-HS3

d) HS2-HS2
e) HS2-HS3
f) HS2-HS1
g) HS3-HS3
f) HS3-HS1
h) HS3-HS2

### Given Observation sequence is ON ON

The possible state sequences are

a) HS1-HS1
b) HS1-HS2
c) HS1-HS3
d) HS2-HS2
e) HS2-HS3
f) HS2-HS1
g) HS3-HS3
f) HS3-HS1
h) HS3-HS2
Probability of a given observation sequence ON ON is $P(ON,ON)$ and is calculated as follows:

Probability of the state sequence HS1 – HS1 is
$$P(HS1 \text{ HS1})=\pi_1 b_1(o_1) a_{11} b_1(o_1)$$

Probability of the state sequence HS1 – HS2 is
$$P(HS1 \text{ HS2})=\pi_1 b_1(o_1) a_{12} b_2(o_1)$$

Probability of the state sequence HS1 – HS3 is
$$P(HS1 \text{ HS3})=\pi_1 b_1(o_1) a_{13} b_3(o_1)$$

Probability of the state sequence HS2 – HS2 is
$$P(HS2 \text{ HS2})=\pi_2 b_2(o_1) a_{22} b_2(o_1)$$

Probability of the state sequence HS2 – HS3 is
$$P(HS2 \text{ HS3})=\pi_2 b_2(o_1) a_{23} b_3(o_1)$$

Probability of the state sequence HS2 – HS1 is
$$P(HS2 \text{ HS1})=\pi_2 b_2(o_1) a_{21} b_1(o_1)$$

Probability of the state sequence HS3 – HS3 is
$$P(HS3 \text{ HS3})=\pi_3 b_3(o_1) a_{33} b_3(o_1)$$

Probability of the state sequence HS3 – HS1 is
$$P(HS3 \text{ HS1})=\pi_3 b_3(o_1) a_{31} b_1(o_1)$$

Probability of the state sequence HS3 – HS2 is
$$P(HS3 \text{ HS2})=\pi_3 b_3(o_1) a_{32} b_2(o_1)$$

Given Observation sequence is OFF OFF
The possible state sequences are
HS4-HS4

Probability of a given observation sequence OFF OFF is $P(OFF,OFF)$ and is calculated as follows:

Probability of the state sequence HS4 – HS4 is
$$P(HS4 \text{ HS4})=\pi_4 b_4(o_2) a_{44} b_4(o_2)$$

For a given observation $O$ we will get the most probable states that our system can reach.

C. Train the Model Parameters $\lambda = (A, B, \pi)$ to Maximize $P(O/\lambda)$ using Baum-Welch Algorithm

$\pi$=starting probability matrix

$B=b_i(O_t)$=observation probability matrix

$A= a_{ij}$=state transition probability matrix
α=foward variable
β=backward variable
P(O|λ)= probability of observation sequence
γ_t(i)=gamma
γ_t(i,j)=di-gamma
For t=1 to T-1, and for i=1 to N so for j = 1 to N the di-gammas can be written in terms of α, β, A and B as

γ_t(i, j) = \frac{α_t(i)a_jb_j(O_{t+1})β_{t+1}(j)}{P(O|λ)}

The γ_t(i) and γ_t(i,j) are related so gamma can be represented

γ_t(i) = \sum_{j=1}^{N} γ_t(i, j)

Given The γ_t(i) and γ_t(i,j) we verify below that the model λ = (A,B,π) can be re-estimated as follows

STEP 1 For i = 1,...,N let

π = π_i = γ_t(i)

STEP 2 For i = 1,...,N and j = 1,...,N

A = a_{ij} = \frac{\sum_{t=1}^{T-1} γ_t(i, j)}{\sum_{t=1}^{T-1} γ_t(i)}

STEP3  For j = 1,..., N and k = 1,..., M

B = b_j(k) = \frac{\sum_{t=1}^{T-1} γ_t(j)}{\sum_{t=1}^{T-1} γ_t(j)}

Table 1: Reliability Parameters of Basic Components on PMU

<table>
<thead>
<tr>
<th>MODULE 1</th>
<th>MODULE 2</th>
<th>MODULE 3</th>
<th>MODULE 4</th>
<th>MODULE 5</th>
<th>MODULE 6</th>
<th>MODULE 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ1=0.4155</td>
<td>λ1=0.1923</td>
<td>λ1=0.1383</td>
<td>λ1=0.0188</td>
<td>λ1=0.2368</td>
<td>λ1=0.0228</td>
<td>λ1=0.2751</td>
</tr>
<tr>
<td>λ2=0.4155</td>
<td>λ2=0.1923</td>
<td>λ2=0.1383</td>
<td>λ2=0.7727</td>
<td>λ2=0.0657</td>
<td>λ2=0.0228</td>
<td>λ2=0.2751</td>
</tr>
<tr>
<td>µ1=673.85</td>
<td>µ1= 547.3</td>
<td>µ1 =438</td>
<td>µ1 =312.88</td>
<td>µ1=365</td>
<td>µ1=17520</td>
<td>µ1=365</td>
</tr>
<tr>
<td>µ2=673.85</td>
<td>µ2 = 547.3</td>
<td>µ2 =438</td>
<td>µ2 =365</td>
<td>µ2 =1460</td>
<td>µ2=17520</td>
<td>µ2=365</td>
</tr>
</tbody>
</table>
\[ \lambda = \frac{\text{number of failure of a component in the given period of time}}{\text{total period of time the component was operating}} \]

\[ \mu = \frac{\text{number of repair of a component in the given period of time}}{\text{total period of time the component was being repaired}} \]

\[ \lambda = \text{failure rate} \quad \mu = \text{repair rate}. \]

Hidden Markov Model for a Two Component System

Fig. 5 shows Hidden Markov Model for a Two Component System with 4 states.

The concept of obtaining Graph from Markov model is well explained in[8]. As we can see from fig 5 State 1 represents the working or healthy state of the system (i.e. both the components of the system are working) if any one of the components stop working but the system is still in working condition then our system reaches State 2 or State 3. When both the components of the system stops working our system comes to State 4 which represent the non working or failure state of the system. These states of a simple four state Markov model are hidden in case of HMM.

Fig. 6 below shows the four state hidden Markov model of each state have two distinct observation Module 1 ie (CT and PT MODULE). Each state has two distinct observation such as ON OFF.
Fig 6 shows the four state hidden Markov model of each state have two distinct observation

The graph of HMM for a two-component system is shown in Fig. 6. The concept of obtaining a graph from a Markov model is well explained in . It is shown in Fig. 6 that state 1 (S1) represents the healthy state of the system when both the components of the system are working. If any one of the components fails to work while the rest of the system is still functional, the system transitions to either state 2 (S2) or state 3 (S3). When both the components of the system fail to work, then the system goes to state 4 (S4), which represents a failure state of the system. These states of a simple four-state Markov model are hidden in the case of an HMM, as shown in Fig. 6. According to the observation states or visible states, the hidden states of the system are predicted.

In the HMM, the hidden states are not visible, but they are probabilistically dependent on each other. The visible observation states are independent of each other, but they are probabilistic functions of the hidden states. Therefore, using the HMM based on the current observation, the future observation and state of the system can be predicted. Eventually, the availability or unavailability of the system at every instance can be computed using HMM.

The HMM shown in Fig. 6 has four hidden states (HS1, HS2, HS3, HS4) and two observation states (O1, O2) per hidden state. HMM can be mathematically represented as

$$\lambda = (A, B, \pi)$$

Given $\lambda_1 = 0.4155$

$\lambda_2 = 0.4155$
\[ \mu_1 = 673.85 \]
\[ \mu_2 = 673.85 \quad \Delta t = 0.00002 \]
so
\[ \lambda_1 = 0.4155 \times 0.00002 = 8.3100 \times 10^{-6} \]
\[ \lambda_2 = 0.4155 \times 0.00002 = 8.3100 \times 10^{-6} \]
\[ \mu_1 = 673.85 \times 0.00002 = 0.0135 \]
\[ \mu_2 = 673.85 \times 0.00002 = 0.0135 \]

\( \lambda = \text{failure rate} \quad \mu = \text{repair rate.} \)

State transition matrix \( A = \)

\[
\begin{bmatrix}
1 - (\lambda_1 + \lambda_2) & \lambda_1 & 0 \\
\mu_1 & 1 - (\mu_1 + \lambda_2) & \lambda_2 \\
\mu_2 & 0 & 1 - (\mu_2 + \lambda_1) \\
0 & \mu_2 & \mu_1 & 1 - (\mu_1 + \mu_2)
\end{bmatrix}
\]

\[ a_{11} = 1 - (\lambda_1 + \lambda_2) = 1.0000 \]

\[
A = \begin{bmatrix}
1.0000 & 8.3100 \times 10^{-6} & 8.3100 \times 10^{-6} & 0 \\
0.0135 & 0.9865 & 0 & 8.3100 \times 10^{-6} \\
0.0135 & 0 & 0.9865 & 8.3100 \times 10^{-6} \\
0 & 0.0135 & 0.0135 & 0.9730
\end{bmatrix}
\]

We have initially assumed the values of the \( B \) Matrix and \( \pi \) Matrix

\[
B = \begin{bmatrix}
0.8 & 0.2 \\
0.6 & 0.4 \\
0.9 & 0.1 \\
0.8 & 0.2
\end{bmatrix}
\]

\[ \pi = [0.3 \ 0.2 \ 0.3 \ 0.2] \]

**MODULE - 2**

Similarly for Module 2 there are four hidden states \( (HS_1, HS_2, HS_3, HS_4) \) and two observation states \( (O1, O2) \) per hidden state. HMM can be mathematically represented as

\[ \lambda = (A, B, \pi) \]

Given
\[ \lambda_1 = 0.1923 \]
\[ \lambda_2 = 0.1923 \]
\[ \mu_1 = 547.5 \]
\[ \mu_2 = 547.5 \quad \Delta t = 0.00002 \]

\[ \lambda_1 = 0.4155 \times 0.00002 = 8.3100 \times 10^{-6} \]
\[ \lambda_2 = 0.4155 \times 0.00002 = 8.3100 \times 10^{-6} \]
\[ \mu_1 = 673.85 \times 0.00002 = 0.0135 \]
\[ \mu_2 = 673.85 \times 0.00002 = 0.0135 \]

\( \lambda = \text{failure rate} \quad \mu = \text{repair rate.} \)
So \[ \lambda_1 = 0.1923 \times 0.00002 = 3.8460 \times 10^{-6} \]
\[ \lambda_2 = 0.1923 \times 0.00002 = 3.8460 \times 10^{-6} \]
\[ \mu_1 = 547.5 \times 0.00002 = 0.0109 \]
\[ \mu_2 = 547.5 \times 0.00002 = 0.0109 \]

\[ \lambda = \text{failure rate} \quad \mu = \text{repair rate}. \]
\[ a_{11} = 1 - (\lambda_1 + \lambda_2) = 1.0000 \]

State transition matrix \( A \) is:

\[
A = \begin{bmatrix}
1 - (\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 & 0 \\
\mu_1 & 1 - (\mu_1 + \lambda_2) & 0 & \lambda_2 \\
\mu_2 & 0 & 1 - (\mu_2 + \lambda_1) & \lambda_1 \\
0 & \mu_2 & \mu_1 & 1 - (\mu_1 + \mu_2)
\end{bmatrix}
\]

Given \[ \lambda_1 = 0.1383 \]
\[ \lambda_2 = 0.1383 \]
\[ \mu_1 = 438 \]
\[ \mu_2 = 438 \quad \Delta t = 0.00002 \]
so \[ \lambda_1 = 0.1383 \times 0.00002 = 2.7660 \times 10^{-6} \]
\[ \lambda_2 = 0.1383 \times 0.00002 = 2.7660 \times 10^{-6} \]
\[ \mu_1 = 438 \times 0.00002 = 0.0088 \]
\[ \mu_2 = 438 \times 0.00002 = 0.0088 \]

We have initially assumed the values of the \( B \) Matrix and \( \Pi \) Matrix

\[
B = \begin{bmatrix}
0.8 & 0.2 \\
0.6 & 0.4 \\
0.9 & 0.1 \\
0.8 & 0.2
\end{bmatrix}
\]
and \( \Pi = [0.3 \ 0.2 \ 0.3 \ 0.2] \)

**MODULE -3**

Similarly for Module-3, there are four hidden states \( (HS_1, HS_2, HS_3, HS_4) \) and two observation states \( (O1, O2) \) per hidden state. HMM can be mathematically represented as

\[ \lambda = (A, B, \pi) \]

Given \[ \lambda_1 = 0.1383 \]
\[ \lambda_2 = 0.1383 \]
\[ \mu_1 = 438 \]
\[ \mu_2 = 438 \quad \Delta t = 0.00002 \]
\[ \lambda = \text{failure rate} \quad \mu = \text{repair rate}. \]
\[ a_{11} = 1 - (\lambda_1 + \lambda_2) = 1.0000 \]

State transition matrix \( A = \)
\[
\begin{bmatrix}
1 - (\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 & 0 \\
\mu_1 & 1 - (\mu_1 + \lambda_2) & 0 & \lambda_2 \\
\mu_2 & 0 & 1 - (\mu_2 + \lambda_1) & \lambda_1 \\
0 & \mu_2 & \mu_1 & 1 - (\mu_1 + \mu_2) \\
\end{bmatrix}
\]

\[
A = \\
\begin{bmatrix}
1.0000 & 2.7666e-006 & 2.7666e-006 & 0 \\
0.0088 & 0.9824 & 0 & 2.7666e-006 \\
0.0088 & 0 & 0.9912 & 2.7666e-006 \\
0 & 0.0088 & 0.0088 & 0.9824 \\
\end{bmatrix}
\]

We have initially assumed the values of the B Matrix and PI Matrix

\[
B = \begin{bmatrix}
0.8 \\
0.6 \\
0.9 \\
0.8 \\
\end{bmatrix} \quad \text{and} \quad \text{PI} = \begin{bmatrix}
0.3 & 0.2 & 0.3 & 0.2 \\
\end{bmatrix}
\]

Problem 1: Given a model \( \lambda = (A,B,\pi) \) and observation sequence \( O \), find \( P(O|\lambda) \). The probability of given observation sequence is obtained for the individual modules of PMU and PMU as a whole using forward algorithm.

**Results of Problem 1**

**MODULE 1**

Observation sequence = [1,1]

Forward algorithm

STEP 1 \( \alpha_1 (i) = \pi_i b_i(O_1) \) for \( i = 1,\ldots,N \)

for \( t = 1,2,\ldots,T \) and \( i=1,\ldots,N \)

Where the sum is from \( i = 1 \) to \( N \)

STEP 1 \( \alpha_1 (i) = \pi_i b_i(O_1) \) for \( i = 1,\ldots,N \)

\[ \alpha_1 (1) = \pi_1 b_1(O_1) = 0.2400 \]
\[ \alpha_1 (2) = \pi_2 b_2(O_1) = 0.1200 \]
\[ \alpha_1 (3) = \pi_3 b_3(O_1) = 0.2700 \]
\[ \alpha_1 (i) = \pi_i b_4(O_1) = 0.1600 \]
STEP 2 \[ a_t(i) = \sum_{j=1}^{m} \alpha_{t+1}(j) a_{ij} b_j(O_i) \]

for \( i = 1, \ldots, N \) using step-2 we can find \( \alpha_2(1), \alpha_2(2), \alpha_2(3), \alpha_2(4) \)

STEP 3

\[ P(O|\lambda) = \sum_{i=1}^{N} \alpha_i = \alpha_2(1) + \alpha_2(2) + \alpha_2(3) + \alpha_2(4) \]

Alpha matrix for the modules 1, 2, 3, 4, 5, 6 and 7 is as follows:

MODULE-1
alpha matrix ( \( \alpha_1 \) )

\[
\begin{bmatrix}
0.2400 & 0.1200 & 0.1600 & 0.1600 \\
0.1245 & 0.0934 & 0.1401 & 0.1245
\end{bmatrix}
\]

MODULE-2
alpha matrix ( \( \alpha_2 \) )

\[
\begin{bmatrix}
0.2400 & 0.1200 & 0.1600 & 0.1600 \\
0.1252 & 0.0939 & 0.1409 & 0.1252
\end{bmatrix}
\]

MODULE-3
alpha matrix ( \( \alpha_3 \) )

\[
\begin{bmatrix}
0.2400 & 0.1200 & 0.2700 & 0.1600 \\
0.1257 & 0.0943 & 0.1415 & 0.1257
\end{bmatrix}
\]

MODULE-4
alpha matrix ( \( \alpha_4 \) )

\[
\begin{bmatrix}
0.2400 & 0.1200 & 0.2700 & 0.1600 \\
0.1263 & 0.0947 & 0.1420 & 0.1263
\end{bmatrix}
\]

MODULE-5
alpha matrix ( \( \alpha_5 \) )

\[
\begin{bmatrix}
0.2400 & 0.1200 & 0.2700 & 0.1600 \\
0.1257 & 0.0943 & 0.1387 & 0.1233
\end{bmatrix}
\]

MODULE-6
alpha matrix ( \( \alpha_6 \) )
So find $P(O/\lambda)$ and Availability of all module using observation sequence.

So unavailability = 1 - Availability

Availability and Unavailability of components of PMU shown in below table.

**Table - 2 Availability And Unavailability Of Components Of Pmu For 4 States Hidden Markov Model**

<table>
<thead>
<tr>
<th>Module</th>
<th>Availability</th>
<th>Unavailability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>0.6033</td>
<td>0.3967</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.6065</td>
<td>0.3935</td>
</tr>
<tr>
<td>$M_3$</td>
<td>0.6091</td>
<td>0.3909</td>
</tr>
<tr>
<td>$M_4$</td>
<td>0.5974</td>
<td>0.4026</td>
</tr>
<tr>
<td>$M_5$</td>
<td>0.6116</td>
<td>0.3884</td>
</tr>
<tr>
<td>$M_6$</td>
<td>0.8145</td>
<td>0.1855</td>
</tr>
<tr>
<td>$M_7$</td>
<td>0.6110</td>
<td>0.389</td>
</tr>
</tbody>
</table>

In Table 2 we can conclude that GPS receiver having low availability compared to other modules therefore parameter of GPS receiver should be considered properly from design point of view

Problem 2: Given $\lambda = (A, B, \pi)$ and $O$, find an optimal state sequence of each module passes through is found

**Results of Problem 2**

Parameters ($\beta$eta & $\gamma$ama Matrix) of a four state hidden Markov model

**MODULE 1**

Observation sequence = [1,1]

STEP 1 Let $\beta_T(i) = 1$ for $i = 1, \ldots, N$

for $t = T-1, T-2, \ldots, 1$ and $i = 1, \ldots, N$, let

Step 1 $\beta_T(i) = 1$

$\beta_2(1) = 1$
β2(2) = 1
β2(3) = 1
β4(4) = 1

STEP 2  \( \beta_t(i) = \sum_{j=1}^{M} a_{ij} b_j(O_{t+1}) \beta_{t+1}(j) \)

Using step-2 we can find \( \beta_1(1), \beta_1(2), \beta_1(3), \beta_1(4) \)

Beta matrix for the modules 1, 2, 3, 4, 5, 6 and 7 is as follows:

Beta matrix (\( \beta_1 \))

\[
\begin{bmatrix}
0.8203 & 0.14230 & 2.3216 & 3.1000 \\
1.0000 & 1.0000 & 1.0000 & 1.0000
\end{bmatrix}
\]

Beta matrix (\( \beta_2 \))

\[
\begin{bmatrix}
0.8164 & 1.4185 & 2.3174 & 3.1000 \\
1.0000 & 1.0000 & 1.0000 & 1.0000
\end{bmatrix}
\]

Beta matrix (\( \beta_3 \))

\[
\begin{bmatrix}
0.8132 & 1.4097 & 2.3088 & 3.0947 \\
1.0000 & 1.0000 & 1.0000 & 1.0000
\end{bmatrix}
\]

Beta matrix (\( \beta_4 \))

\[
\begin{bmatrix}
0.8104 & 1.4124 & 2.3118 & 3.1009 \\
1.0000 & 1.0000 & 1.0000 & 1.0000
\end{bmatrix}
\]

Beta matrix (\( \beta_5 \))

\[
\begin{bmatrix}
0.8307 & 1.4496 & 2.3292 & 3.1000 \\
1.0000 & 1.0000 & 1.0000 & 1.0000
\end{bmatrix}
\]

Beta matrix (\( \beta_6 \))

\[
\begin{bmatrix}
1.3256 & 1.9957 & 2.8606 & 3.1000 \\
1.0000 & 1.0000 & 1.0000 & 1.0000
\end{bmatrix}
\]

Beta matrix (\( \beta_7 \))

\[
\begin{bmatrix}
0.8110 & 1.4124 & 2.3117 & 3.1000 \\
1.0000 & 1.0000 & 1.0000 & 1.0000
\end{bmatrix}
\]

\( \alpha = \) forward variable and \( \beta = \) backward variable

\( P(O|\lambda) = \) probability of observation sequence
\[ \gamma_i(i) = \frac{\alpha_i(i) \cdot \beta_i(i)}{P(O|\lambda)} \quad t=1 \]

\[ \gamma_1(1) = \alpha_1(1) \cdot \beta_1(1) / P(O|\lambda) \quad i=1 \]

\[ \gamma_1(2) = \alpha_1(2) \cdot \beta_1(2) / P(O|\lambda) \quad i=2 \]

\[ \gamma_1(3) = \alpha_1(3) \cdot \beta_1(3) / P(O|\lambda) \quad i=3 \]

\[ \gamma_1(4) = \alpha_1(4) \cdot \beta_1(4) / P(O|\lambda) \quad i=4 \]

Similarly, t=2 we can find \( \gamma_2(1), \gamma_2(2), \gamma_2(3), \gamma_2(4) \) of the modules 1, 2, 3, 4, 5, 6, and 7 is as follows:

Gamma matrix (\( \gamma_1 \))

\[
\begin{bmatrix}
0.4079 & 0.3538 & 1.2988 & 1.0277 \\
0.2581 & 0.1935 & 0.2903 & 0.2581
\end{bmatrix}
\]

Gamma matrix (\( \gamma_2 \))

\[
\begin{bmatrix}
0.4038 & 0.3508 & 1.2896 & 1.0223 \\
0.2581 & 0.1935 & 0.2903 & 0.2581
\end{bmatrix}
\]

Gamma matrix (\( \gamma_3 \))

\[
\begin{bmatrix}
0.4005 & 0.3472 & 1.2793 & 1.0162 \\
0.2581 & 0.1935 & 0.2903 & 0.2581
\end{bmatrix}
\]

Gamma matrix (\( \gamma_4 \))

\[
\begin{bmatrix}
0.3975 & 0.3464 & 1.2758 & 1.0141 \\
0.2581 & 0.1935 & 0.2903 & 0.2581
\end{bmatrix}
\]

Gamma matrix (\( \gamma_5 \))

\[
\begin{bmatrix}
0.4079 & 0.3538 & 1.2988 & 1.0277 \\
0.2581 & 0.1935 & 0.2903 & 0.2581
\end{bmatrix}
\]

Gamma matrix (\( \gamma_6 \))

\[
\begin{bmatrix}
2.1438 & 1.6137 & 5.2045 & 3.3422 \\
0.2581 & 0.1935 & 0.2903 & 0.2581
\end{bmatrix}
\]

Gamma matrix (\( \gamma_7 \))

\[
\begin{bmatrix}
0.3982 & 0.3468 & 1.2770 & 1.0148 \\
0.2581 & 0.1935 & 0.2903 & 0.2581
\end{bmatrix}
\]
Table 3: Computation of Optimal State Sequence For M1

<table>
<thead>
<tr>
<th>Observation Sequence</th>
<th>Possible States Sequence</th>
<th>Probability Value</th>
<th>Optimal State Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ON OFF)</td>
<td>P(HS1 HS4)</td>
<td>0</td>
<td>P(HS3 HS4)</td>
</tr>
<tr>
<td></td>
<td>P(HS2 HS4)</td>
<td>1.9944e-007</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS3 HS4)</td>
<td>4.4874e-007</td>
<td></td>
</tr>
<tr>
<td>(OFF, ON)</td>
<td>P(HS4 HS2)</td>
<td>3.2400e-004</td>
<td>P(HS4 HS3)</td>
</tr>
<tr>
<td></td>
<td>P(HS4 HS1)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS4 HS3)</td>
<td>4.8600e-004</td>
<td></td>
</tr>
<tr>
<td>(OFF OFF)</td>
<td>P(HS4 HS4)</td>
<td>0.0078</td>
<td>P(HS4 HS4)</td>
</tr>
<tr>
<td>(ON ON)</td>
<td>P(HS1 HS1)</td>
<td>0.1920</td>
<td>P(HS3 HS3)</td>
</tr>
<tr>
<td></td>
<td>P(HS1 HS2)</td>
<td>1.1966e-006</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS1 HS3)</td>
<td>1.7950e-006</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS2 HS2)</td>
<td>0.0710</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS2 HS3)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS2 HS1)</td>
<td>0.0013</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS3 HS3)</td>
<td>0.2397</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS3 HS1)</td>
<td>0.0029</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P(HS3 HS2)</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

From TABLE 3 it can be observed that for an observation sequence ON OFF, the optimal state sequence is HS3-HS4; for an observation sequence OFF ON, the optimal state sequence is HS4-HS3; for an observation sequence OFF OFF, the optimal state sequence is HS4-HS4; for an observation sequence ON ON, the optimal state sequence is HS3-HS3; Similarly we can find for M2, M3, M4, M5, M6, M7

Finally, the probability of PMU for different optimal observation sequence is as follows:

PMU

\[
P(\text{ON OFF}) = 4.4874e-007 \times 2.0768e-007 \times 1.4936e-007 \times 3.7090e-007 \times 2.5574e-007 \times 2.4624e-008 \times 2.9711e-007 = 9.6595e-048 \\
P(\text{OFF ON}) = 4.8600e-004 \times 3.9240e-004 \times 3.1680e-004 \times 3.1680e-004 \times 0.0070 \times 0.0126 \times 2.6280e-004 = 4.4364e-022 \\
P(\text{OFF OFF}) = 0.0078 \times 0.0078 \times 0.0079 \times 0.0079 \times 0.0077 \times 0.0024 \times 0.0079 = 5.5434e-016 \\
P(\text{ON ON}) = 0.2397 \times 0.2404 \times 0.2409 \times 0.2415 \times 0.2359 \times 0.1920 \times 0.2412 = 3.6624e-005
\]
<table>
<thead>
<tr>
<th>Probable state sequence</th>
<th>Optimal state sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ON OFF)</td>
<td>9.6595e-048</td>
</tr>
<tr>
<td>(OFF ON)</td>
<td>4.4364e-022</td>
</tr>
<tr>
<td>(OFF OFF)</td>
<td>5.5434e-016</td>
</tr>
<tr>
<td>(ON ON)</td>
<td>3.6624e-005</td>
</tr>
</tbody>
</table>

Problem 3: Given an observation sequence O and N and M, finds the re-estimated model $\lambda = (A, B, \pi)$ that maximizes probability of O.

$\pi$=starting probabilities matrix

$b_i(O_t)$=observation probabilities matrix

$A=$ a$_{ij}$=state transition probabilities matrix

(γ)- di-gamma

Where sum is from j = 1 to N

Given di-gammas (γ) and gamma (γ)

$$\gamma(t, j) = \frac{\alpha_t(i) a_{ij} b_j(O_{t+1}) \beta_{t+1}(j)}{P(O|\lambda)}$$

$$\gamma(i) = \sum_{j=1}^{N} \gamma_t(i, j)$$

STEP 1 For $i = 1,...,N$ let

$\pi = \pi_i = \gamma_t(i)$

STEP 2 For $i = 1,...,N$ and $j = 1,...,N$

$$A = a_{ij} = \frac{\sum_{t=1}^{T-1} \gamma_t(i, j)}{\sum_{t=1}^{T-1} \gamma_t(i)}$$

STEP 3 For $j = 1,...,N$ and $k = 1,...,M$
\[ B = b_j(k) = \frac{\sum_{r=1}^{T-k} \gamma_r(j)}{\sum_{j=1}^{T} \gamma_r(j)} \]

for which \( O_t = k \) are counted in numerator.

Results of Problem 3
Re-estimation of the parameters of a four state HMM,

MODULE 1
Observation sequence = [1,1]

\[ \text{gamma} = [0.3978 \ 0.1492 \ 0.5035 \ 0.2653] \]

\[ \text{digamma} = \begin{bmatrix} 0.3978 & 0.0000 & 0.0000 & 0 \\ 0.0036 & 0.1457 & 0 & 0.0000 \\ 0.0054 & 0 & 0.4981 & 0.0000 \\ 0 & 0.0020 & 0.0046 & 0.2586 \end{bmatrix} \]

\[ \text{A} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0 \\ 0.0239 & 0.9684 & 0 & 0.0000 \\ 0.0107 & 0 & 0.9905 & 0.0000 \\ 0 & 0.0076 & 0.0172 & 0.9750 \end{bmatrix} \]

\[ \text{B} = \begin{bmatrix} 0.1514 & 0.1417 \\ 0.1503 & 0.1406 \\ 0.1464 & 0.1373 \\ 0.1445 & 0.1356 \end{bmatrix} \]

\[ \text{PI} = [0.3978 \ 0.1492 \ 0.5035 \ 0.253 ] \]

MODULE 2
Observation sequence = [1,1]

\[ \text{gamma} = [0.3978 \ 0.1492 \ 0.5008 \ 0.2639] \]

\[ \text{digamma} = \begin{bmatrix} 0.3957 & 0.0000 & 0.0000 & 0 \\ 0.0029 & 0.1456 & 0 & 0.0000 \\ 0.0043 & 0 & 0.4965 & 0.0000 \\ 0 & 0.0016 & 0.0037 & 0.2585 \end{bmatrix} \]
\[
A = \begin{bmatrix}
1.0000 & 0.0000 & 0.0000 & 0 \\
0.0139 & 0.9744 & 0 & 0.0000 \\
0.0086 & 0 & 0.9923 & 0.0000 \\
0 & 0.0061 & 0.0139 & 0.9798 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.1514 & 0.1417 \\
0.1503 & 0.1407 \\
0.1464 & 0.1373 \\
0.1446 & 0.1356 \\
\end{bmatrix}
\]

\[
\Pi = \begin{bmatrix}
0.3957 & 0.1484 & 0.5008 & 0.2639 \\
\end{bmatrix}
\]

MODULE 3
\[
\gamma = \begin{bmatrix}
0.3940 & 0.1478 & 0.4987 & 0.2627 \\
\end{bmatrix}
\]

\[
\text{digamma} = \begin{bmatrix}
0.3940 & 0.0000 & 0.0000 & 0 \\
0.0023 & 0.1454 & 0 & 0.0000 \\
0.0035 & 0 & 0.4952 & 0.0000 \\
0 & 0.0013 & 0.0029 & 0.2584 \\
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
1.0000 & 0.0000 & 0.0000 & 0 \\
0.0157 & 0.9791 & 0 & 0.0000 \\
0.0070 & 0 & 0.9938 & 0.0000 \\
0 & 0.0050 & 0.0112 & 0.9837 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.1516 & 0.1418 \\
0.1504 & 0.1408 \\
0.1466 & 0.1374 \\
0.1446 & 0.1357 \\
\end{bmatrix}
\]

\[
\text{PI matrix} = 0.3940 \ 0.1478 \ 0.4987 \ 0.2627
\]

MODULE 4
\[
\gamma = \begin{bmatrix}
0.3924 & 0.1472 & 0.4967 & 0.2616 \\
\end{bmatrix}
\]

\[
\text{digamma} = \begin{bmatrix}
0.3924 & 0.0000 & 0.0000 & 0 \\
0.0016 & 0.1455 & 0 & 0.0000 \\
0.0029 & 0 & 0.4938 & 0.0000 \\
0 & 0.0011 & 0.0021 & 0.2584 \\
\end{bmatrix}
\]
\[
A = \begin{bmatrix}
1.0000 & 0.0000 & 0.0000 & 0 \\
0.0112 & 0.9851 & 0 & 0.0000 \\
0.0058 & 0 & 0.9949 & 0.0000 \\
0 & 0.0041 & 0.0080 & 0.9879
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.1516 & 0.1418 \\
0.1504 & 0.1408 \\
0.1466 & 0.1374 \\
0.1447 & 0.1357
\end{bmatrix}
\]

\[
\Pi = [0.3924 \ 0.1472 \ 0.4967 \ 0.2616]
\]

MODULE 5

\[
\gamma = [0.4018 \ 0.1507 \ 0.5085 \ 0.2670]
\]

\[
digamma = \begin{bmatrix}
0.4018 & 0.0000 & 0.0000 & 0 \\
0.0020 & 0.1487 & 0 & 0.0000 \\
0.0018 & 0 & 0.4966 & 0.0000 \\
0 & 0.0044 & 0.0025 & 0.2608
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
1.0000 & 0.0000 & 0.0000 & 0 \\
0.0129 & 0.9828 & 0 & 0.0000 \\
0.0231 & 0 & 0.9794 & 0.0000 \\
0 & 0.0165 & 0.0094 & 0.9770
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0.1514 & 0.1417 \\
0.1503 & 0.1466 \\
0.1464 & 0.1373 \\
0.1445 & 0.1356
\end{bmatrix}
\]

\[
\Pi = [0.4018 \ 0.1507 \ 0.5085 \ 0.2670]
\]

MODULE 6

\[
\gamma = [1.2938 \ 0.5438 \ 1.6167 \ 0.8969]
\]

\[
digamma = \begin{bmatrix}
1.2938 & 0.0000 & 0.0000 & 0 \\
0.2706 & 0.2610 & 0 & 0.0000 \\
0.4717 & 0 & 1.1344 & 0.0000 \\
0 & 0.1778 & 0.4252 & 0.2585
\end{bmatrix}
\]
A = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0 \\ 0.5090 & 0.4422 & 0 & 0.0000 \\ 0.2937 & 0 & 0.7263 & 0.0000 \\ 0 & 0.2064 & 0.5203 & 0.2882 \end{bmatrix}

B = \begin{bmatrix} 0.1420 & 0.1328 \\ 0.1408 & 0.1318 \\ 0.1373 & 0.1287 \\ 0.1354 & 0.1271 \end{bmatrix}

P_l = [1.2938 \ 0.5438 \ 1.6167 \ 0.8969]

MODULE 7

\gamma = [0.3928 \ 0.1473 \ 0.4972 \ 0.2619]

\digamma = \begin{bmatrix} 0.3928 & 0.0000 & 0.0000 & 0 \\ 0.0019 & 0.1454 & 0 & 0.0000 \\ 0.0029 & 0 & 0.4943 & 0.0000 \\ 0.0011 & 0.0024 & 0.2584 & 0 \end{bmatrix}

A = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0 \\ 0.0129 & 0.9828 & 0 & 0.0000 \\ 0.0058 & 0 & 0.9949 & 0.0000 \\ 0 & 0.0041 & 0.0093 & 0.9865 \end{bmatrix}

B = \begin{bmatrix} 0.1516 & 0.1418 \\ 0.1504 & 0.1408 \\ 0.1466 & 0.1374 \\ 0.1446 & 0.1357 \end{bmatrix}

P_l = [0.3928 \ 0.1473 \ 0.4972 \ 0.2619]

CONCLUSION

For efficient operation, monitoring and control of the largest and most complex machine such as power system using wide area measurement system (WAMS), the reliable operation of phasor measurement unit (PMU) is extremely essential. For such a complex system, Hidden Markov model (HMM) seems to be promising for computing the probability of given observation sequence is obtained.
for the individual modules and PMU as a whole using forward and. Secondly find backward algorithm and the optimal state sequence each module found. Thirdly the parameters of the hidden Markov model are re-estimated using Baum-Welch algorithm. it has been observed that the availability of GPS Receiver is lower compared to other modules thus ,it is considered as the most sensitive component of PMU and should be given proper attention to it from design point of view for reliable operation of PMU.

REFERENCES

Amaresh Choudhury received the B.tech Eng. from Sanjaya memorial institute of technology, Berhampur odisha, in 2011 and M.tech Eng.degrees in electrical engineering from National Institute of Science and Technology, Berhampur, odisha in 2014; He was guest Lecturer in Parala maharaja engineering college from 01.09.2014 to 30.11.2014, Berhampur, Ganjam .His major research interests include wide-area Monitoring system and its application in power system.