Soft g*s Closed Sets in Soft Topological Spaces

Authors

M. Suraiya Begum¹, M. Sheik John²

Department of Mathematics, NGM College, Pollachi, Tamilnadu, India

Email- suraiya0291@gmail.com, sheikjohn@gmail.com

ABSTRACT

In 1991, Molodstov [9] introduced the concept of soft set theory to deal with uncertainty. Modern topology depends strongly on the ideas of soft set theory. This prompted us to introduce and study soft g*s closed set, soft g*s open set in soft topological spaces also some related properties concerning these sets are obtained.

Keywords- Soft topological space, Soft g*s closed set, Soft g*s open set, Soft Q set.

INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. It was first proposed by Molodtsov [9] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. In 2010 Muhammad Shabir, Munazza Naz [11] defined the theory of soft topological space over an initial universe with a fixed set of parameters. The investigation of generalized closed sets by Levine [5] has led to several new and interesting concepts like covering properties and separation axioms. The notion of generalized closed sets in soft topology was introduced by K. Kannan [4] in 2012. In this paper we define soft g*s closed, soft g*s open and soft Q set in a soft topological space. We also investigate related properties of these sets and compared their properties with other existing soft closed sets in soft topological spaces.

2. PRELIMINARIES

Throughout this paper, (X, τ, E) represents a nonempty soft topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset (A, E) of X, the closure, the interior and the complement of (A, E) are denoted by cl(A, E), int(A, E) and (A, E)c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1. [7] Let X be an initial universal set and E be the set of parameters. Let P(X) denote the power set of X and A ⊆ E. The pair (F, A) is called a soft set over X, where F is a mapping given by F : A → P(X).

Definition 2.2. [3] A soft set (F,E) over X is said to be
i) A null soft set, denoted by φ, if ∀ e ∈ E, F(e) = φ.
ii) An absolute soft set, denoted by X, if ∀ e ∈ E, F(e) = X.

The soft sets (F,E) over an universe X in which all the parameters of the set E are same is a family of soft sets, denoted by SS(X,E).

Definition 2.3. [3]Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if
i) X belongs to τ.
ii) The union of any number of soft sets in \( \tau \) belongs to \( \tau \).

iii) The intersection of any two soft sets in \( \tau \) belongs to \( \tau \).

The triplet \((\tilde{X}, \tau, E)\) is called a soft topological space over \( \tilde{X} \) and any member of \( \tau \) is known as soft open set in \( \tilde{X} \). The complement of a soft open set is called soft closed set over \( \tilde{X} \).

**Definition 2.4.** [3] Let \((\tilde{X}, \tau, E)\) be a soft topological space over \( \tilde{X} \) and \((F, E)\) be a soft set over \( \tilde{X} \). Then

i) Soft interior of a soft set \((F, E)\) is defined as the union of all soft open sets contained in \((F, E)\).

Thus \(\text{int}(F, E)\) is the soft largest open soft set contained in \((F, E)\).

ii) Soft closure of a soft set \((F, E)\) is the intersection of all soft closed super sets \((F, E)\). Thus \(\text{cl}(F, E)\) is the smallest soft closed soft set over \( \tilde{X} \) which contains \((F, E)\).

**Definition 2.5.**

A subset \((A, E)\) of a soft topological space \((\tilde{X}, \tau, E)\) is said to be:

1. A soft semi open set [1] if \((A, E) \subseteq \text{cl}(\text{int}(A, E))\) and soft semi-closed if \(\text{int}(\text{cl}(A, E)) \subseteq (A, E)\).

2. A soft pre open set if \((A, E) \subseteq \text{int}(\text{cl}(A, E))\) and soft pre closed if \(\text{cl}(\text{int}(A, E)) \subseteq (A, E)\).

3. An soft \(\alpha\) open set [1] if \((A, E) \subseteq \text{int}(\text{cl}(A, E))\) and an soft \(\alpha\) closed set if \(\text{cl}(\text{int}(A, E)) \subseteq (A, E)\).

4. A soft regular set [3] if \(\text{int}((A, E)) = (A, E)\) and a soft regular soft set if \(\text{cl}(\text{int}(A, E)) = (A, E)\). The intersection of all soft semi-closed (resp. preclosed, \(\alpha\)-closed) sets containing a subset \((A, E)\) of \((\tilde{X}, \tau)\) is called soft semi-closure [6] (resp. soft pre closure, soft \(\alpha\)-closure) of \((A, E)\) and is denoted by \(\text{scl}(A, E)\) (resp. \(\text{pcl}(A, E), \text{cl}_\alpha(A, E)\)). The soft semi-interior of \((A, E)\) is the largest soft semi open set contained in \((A, E)\) and denoted by \(\text{sint}(A, E)\).

**Definition 2.6.** [3] A subset \((F, E)\) of a soft topological space \((\tilde{X}, \tau, E)\) is called soft generalized-semi closed (soft gs closed) if \(\text{scl}(A, E) \subseteq (U, E)\) whenever \((A, E) \subseteq (U, E)\) and \((U, E)\) is soft open in \(\tilde{X}\).

### 3 SOFT g*s CLOSED SETS

**Definition 3.1.** Let \((\tilde{X}, \tau, E)\) be an STS and \((F, E)\) be a subset of \(\tilde{X}\). The set \((F, E)\) is said to be soft g*s closed if \(\text{scl}(F, E) \subseteq (U, E)\) whenever \((F, E) \subseteq (U, E)\) and \((U, E)\) is soft gs-open set.

The complement of soft g*s closed is called soft g*s open.

**Definition 3.2.** Let \((A, E)\) be a soft set in STS. Then soft g*s closure and soft g*s interior of \((A, E)\) are defined as follows:

i) \(\text{sg}^*\text{cl}(A, E) = \cap\{B, E) : (B, E) \text{ is soft g*s closed set and } (A, E) \subseteq (B, E)\}\)

ii) \(\text{sg}^*\text{int}(A, E) = \cup\{(C, E) : (C, E) \text{ is soft g*s open set and } (C, E) \supseteq (A, E)\}\)

**Theorem 3.3.** Union of two soft g*s closed sets in a STS is soft g*s closed.

**Proof:** Let \((F, E)\) and \((G, E)\) be two soft g*s closed sets in \((\tilde{X}, \tau, E)\). Let \((U, E)\) be a soft gs open sets in \((\tilde{X}, \tau, E)\) such that \((F, E) \cap (G, E) \subseteq (U, E) \Rightarrow (F, E) \subseteq (U, E)\) and \((G, E) \subseteq (U, E)\). Again \(\text{scl}(F, E) \subseteq (U, E)\) and \(\text{scl}(G, E) \subseteq (U, E) \Rightarrow \text{scl}(F, E) \cup (G, E) \subseteq (U, E)\) \(\Rightarrow (F, E) \cup (G, E)\) is soft g*s closed.

**Theorem 3.4.** Intersection of two soft g*s open sets in a STS is soft g*s open.

**Proof:** Let \((A, E)\) and \((B, E)\) be two soft g*s open sets \(\Rightarrow (A, E)^c\) and \((B, E)^c\) are soft g*s closed sets \(\Rightarrow (A, E)^c \cup (B, E)^c\) is a soft g*s closed set \(\Rightarrow ((A, E) \cap (B, E))^c\) is soft g*s closed set. Hence \((A, E) \cap (B, E)\) is soft g*s open set.

**Theorem 3.5.** A soft closed set in a STS is a soft g*s closed set but not conversely.

**Proof:** Let \((A, E)\) be a soft closed set and \((U, E)\) be a soft gs open set such that \((A, E) \subseteq (U, E)\). Now \(\text{cl}(A, E) = (A, E)\). Since \((A, E)\) is soft closed, and every soft closed set is soft semi closed, \(\text{scl}(A, E) \subseteq \text{cl}(A, E) = (A, E) \subseteq (U, E)\). Therefore \((A, E)\) is soft g*s closed.
Example 3.6. Let $\tilde{X} = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1,E), (F_2,E), \ldots, (F_7,E)\}$ where $(F_1,E), (F_2,E), \ldots, (F_7,E)$ are soft sets over $\tilde{X}$ defined as follows:

$F_1(e_1) = \{h_1, h_2\}, F_1(e_2) = \{h_2, h_3\}, F_2(e_1) = \{h_1\}, F_2(e_2) = \{h_{1,3}\}, F_3(e_1) = \{h_2, h_3\}, F_3(e_2) = \{h_1\}, F_4(e_1) = \{h_2\}, F_4(e_2) = \{h_{1,3}\}, F_5(e_1) = \{h_1\}, F_5(e_2) = \tilde{X}, F_6(e_1) = \tilde{X}, F_6(e_2) = \{h_1\}, F_6(e_2) = \{h_{1,3}\}$. Then $\tau$ defines a soft topology on $\tilde{X}$ and hence $(\tilde{X}, \tau, E)$ is a soft topological space over $\tilde{X}$. Now a soft set $(B, E)$ in $(\tilde{X}, \tau, E)$ is defined as follows: $B(e_1) = \{h_1\}, B(e_2) = \{h_3\}$. Then, $(B, E)$ is a soft g*s closed set but not soft closed.

Theorem 3.7. Every soft g*s closed set in a STS is soft gs closed but not conversely.

Proof: Let $(A, E)$ be a soft g*s closed set in an STS $(\tilde{X}, \tau, E)$ and $(U, E)$ be a soft open set such that $(A, E) \subseteq (U, E)$. Since every soft open set is soft gs open and by the definition of g*s closed set $\text{scl}(A, E) \subseteq (U, E)$. Thus $(A, E)$ is soft gs closed.

Example 3.8 Let $\tilde{X} = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1,E), (F_2,E), \ldots, (F_7,E)\}$ where $(F_1,E), (F_2,E), \ldots, (F_7,E)$ are soft sets over $\tilde{X}$ defined as follows:

$F_1(e_1) = \{x_1, x_2\}, F_1(e_2) = \{x_1, x_2\}, F_2(e_1) = \{x_2\}, F_2(e_2) = \{x_1, x_3\}, F_3(e_1) = \{x_2, x_3\}, F_3(e_2) = \{x_1\}, F_4(e_1) = \{x_1\}, F_4(e_2) = \{x_1, x_2\}, F_5(e_1) = \{x_1, x_2\}, F_5(e_2) = \tilde{X}, F_6(e_1) = \tilde{X}, F_6(e_2) = \{x_1, x_2\}, F_7(e_1) = \{x_2, x_3\}, F_7(e_2) = \{x_1, x_3\}$. Then $\tau$ defines a soft topology on $\tilde{X}$ and hence $(\tilde{X}, \tau, E)$ is a soft topological space over $\tilde{X}$. Now a soft set $(H, E)$ in $(\tilde{X}, \tau, E)$ is defined as follows: $H(e_1) = \{x_3\}, H(e_2) = \{x_3\}$. Then, $(H, E)$ is a soft g*s closed set but not soft g*s closed.

Theorem 3.9. Every soft semi closed set in a STS is soft g*s closed but not conversely.

Proof: Let $(A, E)$ be a soft semi closed set in an STS $(\tilde{X}, \tau, E)$ and $(U, E)$ be a soft g*s closed set such that $(A, E) \subseteq (U, E)$. Since $(A, E)$ is soft semi closed, $\text{scl}(A, E) = (A, E)$ and so $(A, E)$ is soft g*s closed.

Example 3.10 Let $(\tilde{X}, \tau, E)$ is a soft topological space over $\tilde{X}$ as in Example 3.8. Now a soft set $(H, E)$ in $(\tilde{X}, \tau, E)$ is defined as follows: $H(e_1) = \Phi, H(e_2) = \{x_2\}$. Then, $(H, E)$ is a soft g*s closed set but not soft semi closed.

Theorem 3.11. Every soft g*s closed set in a STS is soft gs closed but not conversely.

Proof: Let $(A, E)$ be soft g*s closed in an STS $(\tilde{X}, \tau, E)$ and $(U, E)$ be soft gs open such that $(A, E) \subseteq (U, E)$. Since every soft semi open set is soft gs open, $\text{slc}(A, E) = (A, E)$ and so $(A, E)$ is soft gs closed.

Example 3.12. Let $(\tilde{X}, \tau, E)$ is a soft topological space over $\tilde{X}$ as in Example 3.6. Then $\tau$ defines a soft topology on $\tilde{X}$. Now a soft set $(A, E)$ in $(\tilde{X}, \tau, E)$ is defined as follows: $A(e_1) = \{x_2\}, A(e_2) = \{x_1\}$. Then $(A, E)$ is a soft g*s closed set but not soft gs closed.

Theorem 3.13. Every soft a closed set in a STS is soft g*s closed but not conversely.

Proof: Let $(A, E)$ be a soft a closed set such that $(A, E) \subseteq (U, E)$ where $(U, E)$ is soft gs open. Since $(A, E)$ is soft a closed, $\text{cl}(\text{int}(\text{cl}(A, E))) \subseteq (A, E)$ and $(A, E) \subseteq (U, E)$, then $\text{cl}(\text{int}(\text{cl}(A, E))) \subseteq (U, E)$ and $(U, E)^c \subseteq \text{int}(\text{cl}(A, E))^c \Rightarrow \text{cl}(\text{cl}(A, E)) \subseteq (U, E)$ and $\text{scl}(A, E) \subseteq \text{cl}(A, E) \Rightarrow \text{scl}(A, E) \subseteq (U, E)$, and so $(A, E)$ is soft g*s closed.

Example 3.14. Let $(\tilde{X}, \tau, E)$ is a soft topological space over $\tilde{X}$ as in Example 3.6. Then $\tau$ defines a soft topology on $\tilde{X}$. Now a soft set $(G, E)$ in $(\tilde{X}, \tau, E)$ is defined as follows: $G(e_1) = \tilde{X}, G(e_2) = \{h_2\}$. Then, $(G, E)$ is a soft g*s closed set but not soft semi closed.

Theorem 3.15. Let $(B, E)$ be a soft g*s closed set in $(\tilde{X}, \tau, E)$ if and only if $\text{scl}(B, E) / (B, E)$ does not contain any non-empty soft gs closed set.

Proof: Necessity: Assume that $(V, E)$ is a soft gs closed subset of $\text{scl}(B, E) \setminus (B, E) \Rightarrow (V, E) \subseteq \text{scl}(B, E)$ and $(B, E) \subseteq \tilde{X} \setminus (V, E)$. Since $\tilde{X} \setminus (V, E)$ is a soft gs open set, $(B, E)$ is soft g*s closed and $\text{scl}(B, E) \subseteq \tilde{X} \setminus (V, E)$. Therefore, $(V, E) \subseteq \text{scl}(B, E) \cap (\tilde{X} \setminus (\text{scl}(B, E))) = \Phi$. Hence $(B, E)$ is a soft g*s closed set but not soft semi closed.
**Sufficiency:** Assume that \( \text{scl}(B, E) \setminus (B, E) \) does not contain any non-empty soft gs closed set. Let \((B, E) \subseteq (U, E)\) and \((U, E)\) is soft gs open. Suppose that \( \text{scl}(B, E) \) is not contained in \((U, E)\), \( \text{scl}(B, E) \subseteq (U, E)\) is a nonempty soft gs closed set of \( \text{scl}(B, E) \setminus (B, E) \) which is a contradiction. Therefore \( \text{scl}(B, E) \subseteq (U, E) \) and hence \((B, E)\) is a soft g*gs closed set.

**Theorem 3.16.** If \((A, E)\) be a soft g*gs closed set in \((\bar{X}, \tau, E)\) such that \((A, E) \subseteq (B, E) \subseteq \text{scl}(A, E)\) then \((B, E)\) is soft g*gs closed in \((\bar{X}, \tau, E)\).

**Proof:** Assume that \((A, E)\) is soft g*gs closed in \((\bar{X}, \tau, E)\) such that \((A, E) \subseteq (B, E) \subseteq \text{scl}(A, E)\). Let \((U, E)\) be a soft gs open set in \((\bar{X}, \tau, E)\) such that \((B, E) \subseteq (U, E)\). By hypothesis, we have \( \text{scl}(A, E) \subseteq (U, E)\). Now, \( \text{scl}(B, E) \subseteq \text{scl}(A, E) \subseteq (U, E)\). Hence \((B, E)\) is g*gs closed in \((\bar{X}, \tau, E)\).

**Theorem 3.17.** If \((A, E) \subseteq \bar{Y} \subseteq \bar{X}\) and \((A, E)\) is soft g*gs closed in \((\bar{X}, \tau, E)\), then \((A, E)\) is soft g*gs closed relative to \(\bar{Y}\).

**Proof:** Given \((A, E) \subseteq \bar{Y} \subseteq \bar{X}\) and \((A, E)\) is soft g*gs closed in \((\bar{X}, \tau, E)\). Let \((A, E) \subseteq \bar{Y} \subseteq (U, E)\), where \((U, E)\) is soft gs open in \(\bar{X}\). Since \((A, E)\) is soft g*gs closed, \((A, E) \subseteq (U, E)\) implies \(\text{scl}(A, E) \subseteq (U, E)\). It follows that \(\bar{Y} \subseteq \text{scl}(A, E) \subseteq \bar{Y} \subseteq (U, E)\). Hence \((A, E)\) is a soft g*gs closed set relative to \(\bar{Y}\).

**Theorem 3.18.** Let \((B, E)\) be a subset of a soft topological space \(\bar{X}\), then the following are equivalent:

1. \((B, E)\) is soft regular open,
2. \((B, E)\) is soft open and soft g*gs closed.

**Proof.** (1) \(\Rightarrow\) (2). Let \((H, E)\) be a soft gs open set in \(\bar{X}\) containing \((B, E)\). Since every soft regular open set is open, \((B, E) \cup \text{int}(\text{cl}(B, E))) \subseteq (B, E) \subseteq (H, E)\). Hence \(\text{scl}(B, E) \subseteq (H, E)\), \((B, E)\) is soft g*gs closed.

(2) \(\Rightarrow\) (1). Since \((B, E)\) is soft open and soft g*gs closed, then \(\text{scl}(B, E) \subseteq (B, E)\) and so \((B, E) \cup \text{int}(\text{cl}(B, E))) \subseteq (B, E)\), but \((B, E)\) is soft open, \(\text{int}(\text{cl}(B, E)) \subseteq (B, E)\). Since every soft open set is soft pre open, we have \((B, E) \subseteq \text{int}(\text{cl}(B, E))\). Therefore \((B, E) = \text{int}(\text{cl}(B, E))\) implies \((B, E)\) is soft regular open.

**Definition 3.19.** A subset \((A, E)\) of a soft topological space \((\bar{X}, \tau, E)\) is said to be a soft Q-set if \(\text{int}(\text{cl}(A, E)) = \text{cl}(\text{int}(A, E))\).

**Theorem 3.20.** If \((B, E)\) is a subset of a soft topological space \(\bar{X}\), the following are equivalent:

1. \((B, E)\) is soft clopen,
2. \((B, E)\) is soft open, a soft Q-set and soft g*gs closed.

**Proof.** (1) \(\Rightarrow\) (2). Since \((B, E)\) is soft clopen, then \((B, E)\) is both soft open and a soft Q-set. Let \((H, E)\) be a soft g-open set in \(\bar{X}\) and \((B, E) \subseteq (H, E)\). Then \((B, E) \cup \text{int}(\text{cl}(B, E)) \subseteq (H, E)\) and so \(\text{scl}(B, E) \subseteq (H, E)\). Hence, \((B, E)\) is g*gs closed in \(\bar{X}\).

(2) \(\Rightarrow\) (1). Hence, by Theorem 3.18, \((B, E)\) is soft regular-open. Since every soft regular-open set is soft open, then \((B, E)\) is soft open. Also, \((B, E)\) is a soft Q-set, then \((B, E)\) is soft closed. Therefore \((B, E)\) is soft clopen.

**REFERENCES**